Towards a Theory of Economic Development* *without* The Owl of Minerva♠

An Outline & a Summary

K. Vela Velupillai
Department of Economics/CIFREM
University of Trento
Via Inama 5
381 00 Trento
Italy

9 June 2009

Dedicated to the memory of Gunnar Myrdal, a reading of whose monumental 3-volume study of the 'Asian Drama', in the exciting, turbulent, year of 1968, was instrumental in shunting me from the study of the sciences to economics.

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* Prepared for an invited presentation at the *International Conference* on **Values and Rules for a New Model of Development** under the sponsorship of the *Centesimus Annus -- Pro Pontifice* initiative. I am greatly indebted to Professor Alberto Quadrio Curzio for inviting, and inspiring, me to make an effort at making the case for the kind of vision I am trying to foster for modelling development.

♠ Perceptive and knowledgeable readers would infer, first, that I have chosen this title with Hegel's sad indictment of philosophy's penchant for being useful only with hindsight - except that one should read that famous penultimate paragraph in the *Preface* to the **Philosophy of Right**, replacing 'philosophy' with 'growth theory' (see next page). Second, there is also the obvious influence of Lord Kaldor's *'Okun Memorial Lectures'*, delivered at Yale University in October, 1983: **Economics without Equilibrium**, (Kaldor, 1985). Kaldor was my first thesis supervisor at Cambridge, just about the time he began his systematic critique of `equilibrium economics', going much beyond his `capital theoretic' criticisms of aggregate neoclassical theories of growth and distribution (cf. for example, Kaldor, 1972).

Abstract

I attempt to outline a strategy for modelling economic development without growth theory[♥], thus returning it to its own noble, independent, traditions Formal growth theory -- whatever its origin -- is shackled to an equilibrium benchmark that, in turn, binds its visions for development modelling within narrow, ahistorical, anti-empirical and completely unrealistic paths. The reasons for this unnaturally `eternal braid' between development models and growth theory are partly due to historical accidents and, mainly, due to a particular kind of mathematical somnambulance. The proximate causes for these infelicities are stated with some substantiation and reasons given for an alternative modelling approach -- together with an outline of model for development without any reliance on any form of growth theory.

♥ This is what is meant with the allusion to '*The Owl of Minerva*' in the title. The point I wish to emphasise is that *formal growth theory,* of any variety, has never succeeded in being useful, except with hindsight - *ex post,* so to speak. Hegel chided philosophy -- read growth theory -- for being so in that famous penultimate paragraph of his *Preface* to **The Philosophy of Right**:

> "One more word about teaching what the world ought to be: Philosophy always arrives too late to do any such teaching. As the *thought* of the world, philosophy appears only in the period after actuality has been achieved and has completed its *formative process*. The lesson of the concept, which necessarily is also taught by history, is that only in the ripeness of actuality does the ideal appear over against the real, and that only then does this ideal comprehend this same real world in its substance and build it up for itself into the configuration of an intellectual realm. When philosophy paints its gray in gray, then a configuration of life has grown old, and cannot be rejuvenated by this gray in gray, but only understood; *the Owl of Minerva takes flight only as the dusk begins to fall*."
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> G.W.F Hegel: **The Philosophy of Right** (first italic in the original), June 25, 1820.

I am, however, not so sure that the German original *Bildungsprozeß* should be translated as 'formative process'. In the context in which I am invoking this Hegelian thought, I think it is better to translate it as 'developing processes'.

§1. A Preamble

> "You will be speaking of <<The Econometric Approach to Development Planning>>. This is the theme of your study week, a theme which seeks to gather together the latest results of a new branch of science, econometry, and to present them to political economists in order to aid them in formulating those plans for a more stable security and for greater development which can contribute so much to the well-being and peace of nations."
> *Pontificia Academia Scientiarum*, 1965, p. xxxvii.

A little under fifty years ago, on October 7–13, 1963, at the *Pontificia Academia Scientiarum*, eighteen outstanding economists presented scientific studies, during a *Study Week* on ***The Econometric Approach to Development Planning*** (*ibid*). A measure of the significance of the economists and econometricians who presented their visions on The Econometric Approach to Development Planning on that occasion can be gauged from the fact the seven of them went on to become Nobel Laureates.

Development Planning was very much a topical issue, both in practical policy terms and formal analytical terms in economic theory. In growth theory, the golden age of golden rules of accumulation had just been heralded and optimal control theory and dynamic programming models were unifying the planners and market adherents on both sides of the cold war. Prime Minister Hayato Ikeda had launched the income doubling plan for a resurgent Japanese Economy. Prasanta Chandra Mahalanobis, the father of the Indian Planning experiments, now irreversibly abandoned, spoke at the Study Week in broad, visionary, enlightened terms on the aims for '*The Social Transformation for National Development*' by means of economic development – the subject to which we have now returned, here, in the same city, under the auspices of the identical august body.

Remarkably, one of the contributions at that Study Week in the Vatican, the one by Tjalling Koopmans: *On the Concept of Optimal Economic Growth* (pp. 225-300), provided the mathematical economic foundations for the kind of growth theory that dominates current orthodoxy. Koopmans revived a noble tradition of growth theory, underpinned also by ethical and political considerations of intergenerational issues, initiated by Frank Ramsey in 1928 and reconsidered by Jan Tinbergen (who was present at *The Study Week*) and Richard Goodwin, in the late 1950s.

Sadly, the ethical considerations that had prompted Frank Ramsey to be wary of discounting the future, in mathematical models related to optimal control – at the time Ramsey wrote it was the mathematics of the calculus of variations that was being applied - seems to have got lost in the intervening years. The 'Euler equation' – and its dynamic programming counterparts, the Bellman equation, the Value Function, and so on – are routinely derived by second year undergraduates, without the slightest ethical blot on their conscience about the discounting of intergenerational welfare.

In an unintended sense, then, at least one – but, in fact, more than one – of the famous contributions to *The Study Week* has been 'immortalized' in the orthodox growth literature of economics and, thereby, inadvertently no doubt, shunted the resurgent development economics to be underpinned by the irrelevant formalities of *The Owl of Minerva*, *blind to ethics*, epistemologies and phenomenological indeterminacies.

My reading and reflections of the contributions to *The Study Week*, and their unintended impact on subsequent research and consolidation of a vision for growth theory and, eventually, also for a crippled theory of development, can be summarised in the following way. Growth theory became formal and narrowly technical and lost its anchoring in the moral sciences and thereby became devoid of philosophical, epistemological and ontological significance. Hence, to the extent that any theory of development was underpinned by growth theory, it shared these deficiencies. More importantly, from an epistemological point of view – and, perhaps, also methodological – the *uncertain*, *undecidable*, *complex*, *incomplete*, *unsolvable* dimensions[i] of development theory and policy were short-circuited by a reliance on trivially applicable mathematics, wholly without significance for the monumental issues that have to be tackled in a theory of development. Above all, the element of humility that accompanies uncertain, tentative, undecidable, comples, incomplete, indeterminate dimensions, has been pawned to a formalization that has undressed the Emperor.

The agenda I set for a theory of development is in the spirit of what was attempted in the decade and a half, between 1943–1958, an 'era' our most recent Nobel Laureate has felicitously (and correctly, in my view) termed the period of High Development Theory (see, section 3, below). I aim to reformulate theorising about development in the Polya-Simon framework of Problem Solving, where economic development is formalized as a

[i] I invoke these concepts in their strict model, proof and recursion theoretic senses.

Combinatorial Decision Problem, thus unshackling it from orthodoxy's senseless adherence to an unrealistic mathematics and a non-applicable, non-empirical, epistemology.

The three pillars on which I try to erect my structures are provided by Polya-Simon, Michael Polanyi and Schumpeter. From Schumpeter, who I consider to be my intellectual grandparent, via Richard Goodwin, I extract a vision and a conceptual underpinning, orthogonal to orthodox economic theory, for theorising about economic development. Michael Polanyi's 'tacit dimension' in knowledge and its epistemologies provide me with the philosophical basis for introducing the purely personal, human, dimension in formalising decision problems in all its rich indeterminacies. Finally, the formalization of economic development as a Combinatorial Decision Problem is inspired by the Polya-Simon methods of human and organisational problem formulations and their algorithmic underpinnings. This latter line of thought and attack allows me to appeal to the mathematics of constructive and computable analysis – a world of mathematics rich in indeterminacies, undecidabilities, unsolvabilites, uncomputabilities and incompleteness.

These formal worlds allow me to return the tradition of theorising about economic development by the phenomenologists, epistemologists and the philosophers, who have a more humane and humble vision of human and institutional possibilities.

§2. An Agenda

> “"By `development,' .. we shall understand only such changes in economic life as are not forced upon it from without but arise *by its own initiative, from within*. ....
> [T]he mere growth of the economy, as shown by the growth of population and wealth, [will not] be designated here as a process of development. ....
> Development in our sense is a distinct phenomenon, entirely foreign to what may be observed in the circular flow or in the tendency towards equilibrium. *It is spontaneous and discontinuous change in the channels of the flow, disturbance of equilibrium, which forever alters and displaces the equilibrium state previously existing*. Our theory of development is nothing but a treatment of this phenomenon and the processes incident to it. "
> *Schumpeter ([1911], 1934), pp. 63-4; italics added*.

Theorising in economic dynamics has always been enriched by imaginative – albeit misleading – metaphors and analogies. From Wicksell's careful use of the 'Rocking Horse' metaphor, incorrectly attributed and inappropriately invoked by Frisch, and utilised by all and sundry working within the real business cycle and newclassical paradigms, all the way through Swan and his *Meccano sets* and, now, Romer with his appeal to a child's *Chemistry* *set*, growth theory – with developmental pretensions - in particular has been replete with them. I shall follow this tradition and invoke three such metaphors for suggesting a way to model economic development without any underpinnings in formal growth theory: the *Jigsaw Puzzle*, *Chess* and the *game of GO*.

§2.1 Schumpeterian Precepts for Modelling Development

Very soon after arriving at the European University Institute for a few happy years of a Professorial stint, not least due to - and because of - the presence of Richard Goodwin in neighbouring Siena, I took the chance to invite him to give us a talk on Schumpeter, which he did under the uncharacteristic -- for the austere scholar he was -- title: *Schumpeter - The Man I Knew*[ii]. In these very personal reflections on Schumpeter there was an almost throwaway remark on a particular modelling metaphor that has, in recent years, been severely `bastardized' by endogenous and evolutionary growth theorists[iii], the notion of 'creative destruction':

> "[I]t came as a great shock to me to find that in the very last paper he ever wrote, before dying in his sleep in his home at Taconic, he said the future of research lay in the study of the records of the great business enterprises – no mention of econometric model building and testing.
> Now after many years, and *in view of the poor results of model building*, I sympathise much more with his point of view and see it as the logical culmination of his own unique contribution. Like Marx he was a student of the morphogenetic nature of capitalism. The economy is not a given structure like von Neumann's model, or a collection of hydrogen atoms, it is an organism perpetually altering its own structure, generating new forms. Unlike most organisms it does not exhibit durable structural stability: it is perhaps best thought of as [being in] a kind of hyper-Darwinian, perpetual evolution. We are so familiar with it, we normally do not realize how remarkable it is. It is not like morphogenesis in animals and plants, where the species is programmed to generate a particular structure, and exhibits structural stability by creating the same form for thousands of years. Rather it is analogous to the much disputed problem of the generation of new species.
> The economy is unsteadily generating new productive structures. In this sense *Schumpeter was profoundly right to reject the elegant new mathematical models: they are the analysis of the behaviour of a given structure. He saw that not only was the economy creatively destroying parts of its given structure, but also that one could not analyze a given structure, ignoring that this cannibalism was going on*."
> *Goodwin, 1989, pp. 107-8; italics added.*

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ii For several years it circulated in a proverbial samizdat existence only as an EUI Discussion Paper which, in those early years of the existence of the department of economics, had a very limited circulation.

iii With the notable exception of three of the pioneers of evolutionary dynamics in economics: Richard Day, Richard Nelson and Sidney Winter.

Coupling this observation with the guidelines in chapter 2 of *The Theory of Economic Development*, summarised in the extensive opening quote of this section, I infer and suggest that an interesting, rich, meaningful set of criteria for theorising about economic development should be in terms of what I have, in other work, come to call '*undecidable dynamics*'. Within the scope and space constraints of this paper I cannot even begin to outline the formal mathematics of undecidable dynamics and will have to resort to self-referencing, however un-humble such an act must be[iv]. The notion and mathematics of undecidable dynamics goes far beyond the ordinary richness of nonlinear dynamics and this is because of the way nonlinear dynamics is coupled – in a formally equivalent, dual, way – to classical recursion theory and, thereby, made amenable to issues of decidability, incompleteness, unsolvability and uncomputabilty (an example of which is given in the penultimate section, below).

§2.2. High Development Theory – an Interregnum

> "It will become apparent that what I identify as `*high development theory*' is essentially the nexus among the external economy/balanced growth debate, the concept of linkages, and the surplus labor doctrine. This theory's golden age began with Rosenstein-Rodan (1943) and more or less ended with Hirschman (1958). Obviously *this nexus does not cover all of what was happening in the field of development economics even at that time, but it is the core of what I believe needs to be recaptured*."
> *Krugman (1993), p.16; italics added.*

The architects of High Development Theory (HDT), in the sense in which Krugman has defined the field, were Rosenstein-Rodan, Hans Singer, Raul Prebisch, Harvey Leibenstein, Richard Nelson, Gunnar Myrdal and Albert Hirschman – but the inclusion of Jan Tinbergen, Ragnar Frisch, Ragnar Nurkse and Arthur Lewis to this pantheon will not be incongruous in any sense at all. That Schumpeter is not included in this list, especially since Hans Singer was seriously motivated by his early studies under him (and Spiethoff and Keynes), and also Nurkse took the Schumpeterian vision seriously, is anomalous. I suspect that this was because Schumpeter's Theory of Economic Development was not interpreted as applying to economies that were still in the early stages of 'capitalistic maturity' (but cf. Laumas, 1962, and the references cited therein, for a dissenting view). Moreover, the appropriation of the Schumpeterian vision by the evolutionary theorists may have also contributed to the lack of attention paid to the applicability – indeed, the richness – of the conceptual framework for the problems of economic underdevelopment.

[iv] I refer, with sincere apologies, to my forthcoming text on *Computable Foundations for Economics* (Routledge, 2009), for detailed discussions of the mathematics of undecidable dynamics.

Be that as it may, summarising the Krugman 'thesis' on HDT in a concise way, we have (all quotations are from Krugman, op.cit, pp. 15-16; italics added):

- "Once upon a time there was a field called development economics – a branch of economics concerned with explaining why some countries are so much poorer than others and prescribing ways for poor countries to become rich. In the field's glory days in the 1950s, the ideas of development were regarded as revolutionary and important and commanded both great intellectual prestige and substantial real-world influence. Moreover, development economics attracted creative minds and was marked by a great deal of intellectual excitement."
- "That field no longer exists ….."
- "And very few economists would now presume to offer grand hypotheses about why poor countries are poor or what they can do about it. In effect, <u>a counterrevolution</u> has swept development economics away.[T]he counterrevolution went too far. [D]uring the 1950s a central core of ideas emerged regarding external economies, strategic complementarity, and economic development that remains intellectually valid and that may continue to have practical applications. This set of ideas … I will refer to as 'high development theory'.

- "High development theory anticipated in a number of ways the cutting edge of modern trade and growth theory. But high development theory was virtually buried, essentially because the founders of development **<u>economics failed to make their points with sufficient analytical clarity to communicate their essence to other economists</u>** … . Recent changes in economics now make it possible to reconsider what the development theorists said and to regain the valuable ideas that have been lost. In other words, *[I] call for a counter-counterrevolution in development theory*".

The essential point that Krugman wants to highlight is quite simple: the economic concepts that were at the core of the theorising exercises of the HDT theorists – *increasing returns to scale*, *complementarity*, *extent of the market*, *multiple equilibria with low level equilibrium traps in poverty*, all classic Smithian themes by any other name – were not amenable to mathematical modelling with the kind of mathematics that was routinely available to the mathematical economist, or even in mathematics. Hence, the theories of the HDT era were not formulated and formalised in a language intelligible to the increasingly mathematised conventional economist. Therefore, the rich content of the HDT theorist was thrown away with the in the proverbial 'baby & bathwater' mode.

Now, with the increasingly sophisticated availability of nonlinear and combinatorial mathematical tools, even to the graduate student in mathematical economics, and given that fields considered adjacent and complementary to development economics – such as industrial organisation theory, international economics and growth theory – have begun to utilise these tools, a time may have come for a reconsideration of the contents of the HDT era.

The catch, however, is this: *equilibrium* rules and *processes* do not! The cardinal Schumpeterian desiderata are relegated to footnotes or afterthoughts or appendices. Of course, a nonlinear model, for example encapsulating the 'Big Push' idea of Rosenstein-Rodan, does entail multiple equilibria – but the *path*, as a *process*, from, say a low level economic equilibrium, to one that is rich in economic possibilities, is added as an *ad hoc* mechanism, not part of the intrinsic dynamics of the model. It may be useful to recall a pungent 'warning' by Schumpeter, on this particular issue:

> "[W]hat we are about to consider is that kind of change arising from within the system *which so displaces its equilibrium point that the new one cannot be reached from the old one by infinitesimal steps*. Add successively as many mail coaches as you please, you will never get a railway thereby."
> *ibid, p. 64; italics in the original.*

This is one of the ways in which the conceptual richness of the HDT period has been emasculated – by forcing them into a straitjacket of a mathematics that was not rich enough to encapsulate the nuances that accompanied the subtle non-economic constraints that Myrdal, Nurkse, Singer, Prebisch, in particular, had in their treatises. Above all, not even the ordinary nonlinear mathematics to which Krugman appeals can begin to encapsulate the kind of *complex dynamics* envisaged in Schumpeter's vision for a theory of development – in the form of '*hyper-Darwinian evolution*[v]'.

Finally, the policy prescriptions, in the form of strategies for economic development, on the basis of these multiple equilibrium, non-algorithmic, pseudo-dynamic, optimization models fall into the standard categories – most notably, concentrate on increasing return sectors, industries or regions, and admonish the politician and the applied economist to recommend the establishment of the appropriate organisational structure so that such 'economies' can be exploited.

That old hobby- horse, reigning during the heyday of optimal planning exercises, 'turnpike theorems', is not even mentioned in footnotes, these days! That is the extent to which

[v] The analogy would be with models of 'hyper-computation', but this is a subject that requires much more serious space for even a simple explication within the narrow scope of this paper.

processes have been removed from the discourse of this kind of attempt at a rejuvenation of HDT, whose *raison d'être* was, in fact, the primacy of (disequilibrium) processes.

§2.3. New Growth Theory – an Aberration

> "All these pretty, polite techniques, made for a well-panelled Board Room and a nicely regulated market, are liable to collapse. At all times the vague panic fears and equally vague and unreasoned hopes are not really lulled, and lie but a little way below the surface."
> *Keynes (1937), p. 215*

I shall summarise one strand of New Growth Theory, simply to indicate, as concisely as possible, why any such framework is wholly inappropriate for providing foundations – or even act as a handmaiden to – for a theory of economic development in particular, for a Schumpeterian vision of economic development. I shall construct my 'strawman' with the material and framework provided by Romer's approach, although I could equally well and easily have resorted to the Lucasian models. Only the details would differ between the two; the fundamental infelicities are identical.

I shall summarise my critique in a schematic form, to highlight the infelicities – at least from the point of view of one who is interested in the kind of theory of economic development advocated by Schumpeter.

Romer's Methodological Credo: 1

- Growth is a general eq$^{\underline{m}}$ process; ∴, a growth theorist must construct a ***dynamic general equilibrium model*** underpinned by explicit specifications of preferences, technology and an equilibrium concept.

- The mathematical tool to be used in the characterization of dynamic ***competitive*** equilibrium models should be the Kuhn-Tucker theorem since it offers a *procedure* for reducing the problem of ***calculating*** competitive equilibria to that of ***solving*** a maximization problem.

- Of all the policy questions concerning growth, the most fundamental is whether there are any policies that an *omniscient*, *omnipotent*, *benevolent social planner* could implement to raise the welfare of all individuals in an economy; i.e., in formal terms, the question is *whether or not equilibria are Pareto optimal*.

- To treat this question seriously, economists must generate a set of models with Pareto optimal & Pareto suboptimal equilibria, s.t., policy questions w.r.t growth facts can be reduced to a choice from such a set.

The 'High Point' of Romer's Methodological Credo: 2

- Given the equivalence between saddle points and competitive equilibria, the economic theoretical implication of the Kuhn-Tucker theorem is:
- ❖ The sufficient conditions of the theorem embody the *First Fundamental Theorem of Welfare Economics*: i.e., ***competitive equilibria are Pareto optimal***
- ➢ The necessary conditions of the theorem imply the Second Fundamental Theorem of Welfare Economics: i.e., for any Pareto Optimally determined quantities, there exists *a price system that decentralizes these quantitites as a competitive equilibrium*

A Romer-inspired formalization of a Representative Agent Growth Theory

Consider a generic constrained maxmization problem Þ:

$$\text{Max } f_0(x) \text{ , s.t } x \in \Omega;\ f_1(x) \geq 0, f_2(x) \geq 0, .., f_m(x) \geq 0$$

To apply the Kuhn-Tucker theorem to ***Þ***,

$f_0(x)$ must be concave and the constraint set, defined by the inequality constraints, $f_i(x) \geq 0$, ∀I=1…m, must be convex.

$\mathcal{L}: \Omega \times \mathbb{R}^m_+ \rightarrow \mathbb{R}$ *by the rule*

$$\mathcal{L}(x, \lambda) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$$

This function is concave convex:

1. When x is held fixed, the function $\mathcal{L}_\lambda(x)$ *that sends* x *to* $\mathcal{L}(x, \lambda)$ is a **convex** function;
2. When λ is held fixed, the function $\mathcal{L}_x(\lambda)$ *that sends* λ *to* $\mathcal{L}(x, \lambda)$ is a **concave** function;
3. i.e., $\mathcal{L}(x, \lambda)$ is a **saddle** function;
4. The essential content of the Kuhn-Tucker theorem is: **saddle points of $\mathcal{L}$ are equivalent to solutions to p**

5 This translates into an equivalence between: **Saddles points for Lagrangians and equilibria for competitive economies**

The Methodological foundations of New Growth Theory:

Given the equivalence between *saddle points* and *competitive equilibria*, the economic theoretical implication of the Kuhn-Tucker theorem is:

The sufficient conditions of the theorem embody the *First Fundamental Theorem of Welfare Economics*: i.e., ***competitive equilibria are Pareto optimal*** $\Longrightarrow$ *The Benevolent Central Planner in action*

The *necessary conditions* of the theorem imply the ***Second Fundamental Theorem of Welfare Economics***: i.e., for any Pareto Optimally determined quantities, **there exists** ***a price system that decentralizes these quantitites as a competitive equilibrium*** $\Longrightarrow$ *The Invisible Hand in action*

Proposition:

Theorising for Economic Development *with* The Owl of Minerva is dangerous – whether in daylight or in darkness.

Remark:

As a matter of fact, I can state and prove a formal theorem encapsulating the slightly playful-looking 'proposition' stated above. Indeed, a series of theorems – or, in true mathematical style, a series of Lemmas, culminating in a theorem – beginning with formal proofs of the non-effectivity and non-constructivity of the two fundamental theorems of welfare economics and going on to a demonstration of the computational complexity of the algorithms used in the implementation of the Kuhn-Tucker theorems. The formal results would make it clear that Romer's (and Lucas's and Prescott's and all their followers and side-kicks) *omniscient, omnipotent, benevolent social planner* would need to invoke nothing less than divine grace and power to *implement efficient policies raise to the welfare of all individuals in an economy*.

Surely, divinity expects more and better from us, as economists!

Let me, however, end with the sane thoughts of a pioneer visionary growth theorist, who was also an 'insider' to the HDT era: Richard Nelson. In his stimulating new book, Nelson (2005), p. 12, he wonders, perceptively:

> "To the extent that formalization of important and previously unformalized understandings about technical change and economic growth defines an important part of the agenda for the new growth theorists, it seems useful to ask why certain ideas have been picked up and formalized and others not.
> ….
> The answer, I believe, is that another part of the agenda of the new growth theory, or a constraint pm that agenda, is to hold the modelling as close as possible to the canons of general equilibrium theory. Romer .. states this explicitly, and the form of the models that have been developed by others suggests that they too hold this as an objective or constraint. However, it certainly seems relevant to think a little about what is gained and what is lost by operating under this constraint."

My attempt at answering just this question, but from the point of view of a theory of economic development, is outlined above.

§2.4. Old – Optimal - Growth Theory – a Reflection

> "A More specific optimality concept is implied in the strictures of Professor Harrod .. and of Frank Ramsey .. against any discounting of future utilities. These authors leave little doubt that they regard only equal weights for the welfare of present and future generations as *ethically defensible*."
> Koopmans, op.cit, p. 226; italics added.

I digress slightly, to return to themes taken up in the Preamble, and to make a link with the previous Vatican meeting which discussed similar issues. For this I present a textbook version of the violation of the ethical spirit and the mathematical warnings of Frank Ramsey – just so that readers can wonder, from yet another point of view, whether there is any point in the whole exercise of formal growth theory as a basis for a theory of economic development. After all, a theory of economic development devoid of ethical concern is almost more meaningless than that proverbial allusion to 'Hamlet without the Prince' – especially in these days of intertemporal concerns that have a global reach.

Frank Ramsey's Two Strictures of 1928 & 1926

- "One point should perhaps be emphasized more particularly; it is assumed that we do ***not*** discount later enjoyments in comparison with earlier ones, a practice which is ***ethically indefensible and arises merely from the weakness of the imagination***; …. "
- "Nothing has been said about degrees of belief when the number of alternatives is ***infinite***. About this I have nothing useful to say, except that ***I doubt if the mind is capable of contemplating more than a finite number of alternatives***. It can consider questions to which an infinite number of answers are possible, but in order to consider the answers it must lump them into a finite number of groups. The difficulty becomes

practically relevant when discussing induction, but even then there seems to me no need to introduce it. We can discuss whether past experience gives a high probability to the sun's rising to-morrow without bothering about what probability it gives to the sun's rising each morning for evermore."

Frank Ramsey: **Foundations – Essays in Philosophy, Logic, Mathematics and Economics**, p 261 & p.85.; emphases added.

Violating the Ramsey Strictures in the Ramsey-Koopmans Textbook Growth Model

Society, whether developed, developing or underdeveloped, is represented by one super-rational, infinitely-lived, economic agent, whose optimal decisions underpin the trajectory of every kind of economy – optimally. The general framework is as follows (I follow, quite literally, the presentation in the well-known advanced macroeconomic textbook by David Romer):

The Household's Utility Function is:

$$U = \int_{t=0}^{\infty} e^{-\rho t} u\left(C(t)\right) \frac{L(t)}{H} dt \quad .... \qquad (1)$$

where: u is the instantaneous utility function, depending on consumption, $C(t)$, at time t; H and $L(t)$ are #s of members in a household and total population, respectively.

Hence, $u\left(C(t)\right)\frac{L(t)}{H}$: the representative household's instantaneous utility at t;

ρ: (subjective) discount rate;

Assume: $u\left(C(t)\right) = \frac{C(t)^{1-\theta}}{1-\theta}$, $\theta > 0$ & ρ-n-(1-θ)g>0 .... (2)

(A constant relative risk aversion utility because the coefficient of relative risk aversion, $\text{-}Cu''(C)/u'(C)$, for this u is θ; i.e., independent of C.)

The Household's Budget Constraint:

$$\int_{t=0}^{\infty} e^{-R(t)} C(t) \frac{L(t)}{H} dt \leq \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} W(t) \frac{L(t)}{H} \quad \ldots \ (3)$$

Rewriting ⇒

$$\frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} \left[W(t) - C(t)\right] \frac{L(t)}{H} dt \geq 0 \quad \ldots \ (4)$$

This is equivalent to:

$$\lim_{s \to \infty} \left[\frac{K(0)}{H} + \int_{t=0}^{s} e^{-R(t)} \left[W(t) - C(t)\right] \frac{L(t)}{H} dt \right] \geq 0$$

Now, the household's capital holdings - wealth - at time s are:

$$\frac{K(s)}{H} = e^{-R(s)} \frac{K(0)}{H} + \int_{t=0}^{s} e^{R(s)-R(t)} \left[W(t) - C(t)\right] \frac{L(t)}{H} dt \quad \ldots \ (5)$$

Hence, the budget constraint can simply be written as:

$$\lim_{s \to \infty} e^{-R(s)} \frac{K(0)}{H} \geq 0 \quad \text{(the no Ponzi game condition)} \quad \ldots \ (6)$$

Defining $c(t) = \frac{C(t)}{A(t)}$ and using $L(t) = L(0)e^{nt}$, we get:

$$U = B \int_{t=0}^{\infty} e^{-\beta t} \frac{c(t)^{1-\theta}}{1-\theta} dt \quad \ldots \ (7)$$

where:

$$B \equiv A(0)^{1-\theta} \frac{L(0)}{H} \quad \& \quad \beta \equiv \rho - n - (1-\theta) g \ \ \& \ \ \beta > 0$$

The budget constraint (3) can be rewritten as:

$$\int_{t=0}^{\infty} e^{-R(t)} c(t) e^{(n+g)t} dt \leq k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t) e^{(n+g)t} dt \quad \ldots \ (8)$$

where, $w(t)$: wage per unit of effective labour &
k : capital per unit of effective labour;

Since, $K(s) \propto k(s) e^{(n+g)s}$, the no Ponzi game version of the budget constraint becomes:

$$\lim_{s \to \infty} e^{R(s)} e^{(n+g)s} k(s) \geq 0 \quad \ldots\ldots\ldots \quad (9)$$

Finally, therefore, the household's optimum decision problem is:
Choose the path of $c(t)$ to maximise lifetime utility (7), subject to the budget constraint (8).

Although I have reproduced, faithfully, an advanced textbook version – ostensibly aimed at graduate students - of the staple diet being fed to the formative minds of economists, this is the kind of violation of explicit *Ramseyan* ethical and mathematical strictures that is also routinely presented to advanced undergraduate students. How, then, can one expect to devise a theory of economic development on such a basis for growth theory?

§. 2.5 A New Development Economics

> "Some three decades ago .. S.N. Eisenstadt wrote that 'historically, modernization is the process of change towards those types of social, economic and political systems that have developed in Westrern Europe and North America from the seventeeth century to the nineteenth.' … [S]uch a view now appears untenable to even its author. And yet where are we to go from here? Clearly, the historians of Japan are at a considerable advantage in the sense that the Japanese are at a considerable advantage in the sense that the Japanese historical tradition itself has long insisted on the singularity of Japanese 'modernization,' deeply rooted in specific institutions but still 'comparable' in some sense to the received Western model. The bottom line, which makes this vision credible, is the 'Japanese 'bank balance'."
> *Subrahmanyam (1998), p. 99*

What I have come to call *The New Development Economics*[vi] is defined and circumscribed by the serendipitously simultaneous appearance of three facts of economic life, at the dawn of the 1980s:

I. The apparent sudden emergence of a new group of inter-related Asian 'Tigers' as fully fledged members of the league of developed nations – Singapore, Hong Kong, Taiwan and Korea (South). This was the first such 'emergence' since the 'miracle' of Japan, many decades earlier, indeed at the dawn of the Showa era.
II. The 'sudden' completion of the UN-ICP program on internationally comparative national income data, in the form of the Penn World Tables.
III. The emergence of New – Endogenous – Growth Theory, going beyond the early Neo-classical models of (optimal) economic growth.

The characteristic feature of the simultaneous – serendipitous – appearance of the above three historical facts, statistical richness and a growth theoretical framework, each reinforcing the validity of the other, was embraced with immense enthusiasm by the economic profession, bent on making the subject truly 'scientific' – i.e., capable of experiments and prediction on a sure footing.

[vi] Not a little influenced by the appearance of New Classical Economics, New Growth Theory, the New Institutional Economics …, and many other such 'New' prefixed fields of economics.

A quarter of a century has elapsed since these serendipitous events dominated and excited the theoretical, applied and policy wings of the economics profession.

I do not find it credible to believe that the three great events listed above have anything to do with each other – either as a matter of fact, history or theory. Let me explain.

Firstly, never in these many years has there ever been the slightest investigation of the methods, sources, consistency and theory of the construction of the data that comes out of the 'Penn World Table' stables. This author took it upon himself to investigate the mathematical foundations of the construction of the index numbers that underpin the Penn World Table data and was appalled to find that the generated numbers have nothing to do with the claimed theoretical foundations for the construction of them.[vii]

Secondly, there is very little evidence that any of the 'Tigers' – and certainly not Japan in its transition from Tokugawa Feudalism to the Meiji Restoration and, then, through the Taisho and Heiwa eras – emerged as powerful economies following the policy precepts of New Growth Theory.

Finally, as I have discussed in the previous sub-sections, nothing in the structure or framework of New Growth Theory can justify it as providing a foundation for a theory of economic development.

§. 2.5 Towards a Combinatorial Formulation of a Theory of Economic Development

> "The key step in understanding economic growth is to think carefully about ideas. This requires careful attention to the meaning of the words that we use and to the *metaphors that we invoke* when we construct models of growth. After addressing these issues, [I describe] …. ways in which ideas can contribute to *economic development*."
> *Romer (1993), p. 63; italics added.*

The background

In two recent publications Paul Romer broached new visions for an understanding of the growth and ***development*** processes in widely differing economic systems (Romer, 1993a, 1993b). Romer points out that the conventional modelling of growth processes are deficient

[vii] In an envisaged expanded version of this paper I shall endeavour to outline and substantiate this claim in some mathematical detail.

in their incorporation of the role and genesis of *ideas*. To rectify this deficiency he proposes an economic definition of *ideas* based, *inter alia*, on a distinction between their *use* and their *production*. These definitions have an evolutionary and algorithmic underpinning to them; moreover, the institutional setting in which ideas are used and produced are also given an evolutionary basis.

For the formal definition of an idea Romer resorts to the imaginative metaphor of toy chemistry sets[viii]. Such sets typically consist of 'a collection of N jars, each containing a different chemical element'. Thus, in a set with N jars there can be at least, say, 2^{K-1} combinations of K elements $(K = 1, \ldots, N)$. If we move from a child's chemistry set to a typical garment factory of a developing country we might find that, say, sewing a shirt entails 52 distinct, sequenced, activities. There are, thus, $52! = 10^{68}$ distinct ordering of the sequences in the preparation of a shirt. Now, as Romer perceptively notes:

> "For any realistic garment assembly operation, almost all the possible sequences for the steps would be wildly impractical, but if even a very small fraction of sequences is useful, there will be many such sequences. It is therefore extremely unlikely that any actual sequence that humans have used for sewing a shirt is the best possible one."
> ibid, p. 69.

Thus:

> "The potential for continued economic growth comes from the vast search space that we can explore. The *curse of dimensionality* [i.e., 2^{K-1}; or $52! = 10^{68}$] *is, for economic purposes, a remarkable blessing*. To appreciate the potential for discovery, one need only consider the possibility that an extremely small fraction of the large number of possible mixtures may be valuable."
> ibid, pp. 68-9[ix]; italics added.

There are some formal problems with these imaginative and interesting observations. First of all, there is the perennial question of the *existence* of a best possible *sequence*. Secondly, even if existence can be proved - in some mathematical sense - it is not clear that it can be discovered and implemented in an operational sense. Thirdly, there *will not* be any feasible way of discovering, *formally*, even the 'extremely small fraction' of sequences that may well be valuable. Finally, even in the unlikely event that all of these issues can satisfactorily be resolved, there is the real question of the *transition* from the currently implemented sequence

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[viii] Economists of my vintage will recall Trevor Swan's brilliant metaphor of *meccano* sets 'to put up a scarecrow.....to keep off the index-number birds and Joan Robinson herself' (Swan, 1956, p. 343).

[ix] This, surely, is a basis for 'learning by doing' emanating from Lundberg's famous 'Horndal effect', made famous by Arrow (1962) and David (1975, ch. 3).

to a 'more valuable region' of the feasible domain. Unless the currently utilized sequence is in the neighbourhood of the 'extremely small valuable fraction' it is unlikely that a transition makes economic sense in the context of a pure growth model with its given institutional background. **The point at which *development* will have to be distinguished from *pure growth* may well be located in this transition manifold, to be somewhat pseudo-mathematical about it.**

These problems need not be faced as squarely within the traditional production theoretic framework with its handmaiden, the *book of blueprints*[x]. In the traditional framework, the well defined concepts of the efficient frontier and concomitant best-practice technologies and so on make most, if not all, of the above issues almost irrelevant. But, by the same token, make it impossible to raise the interesting and important issues that Romer is trying to broach. Romer emphasises time-sequenced processes and, hence, must have something more than the *book of blueprints* metaphor for the repository or encapsulation of ideas.

To return to Romer's *ideas* on *ideas*, the casual empiricism of the above two quotes, underpinned by the metaphor of the child's toy chemistry set and its functions suggests, to him, the analogy of ideas as mixtures; or, as each of the potentially feasible 2^{K-1} mixtures (i.e., each of the $52! = 10^{68}$ ways of sequencing the sewing of a shirt):

> "Within the metaphor of the chemistry set, it is obvious what one means by an idea. Any mixture can be recorded as a bit string, an ordered sequence of 0s and 1s [of length N] .... [A]n idea is the increment in information that comes from sorting some of the bit strings into two broad categories: useful ones and useless ones.............
> When a useful mixture is discovered .... the discovery makes possible the creation of economic value. It lets us combine raw materials of low intrinsic value into mixtures that are far more valuable. Once we have the idea, the process of mixing will require its own [Production Function] (specialized capital and labour). For example, the bit string representing nylon requires a chemical processing plant and skilled workers. Important as these tangible inputs are, it is still the idea itself that permits the resulting increase in value. In this fundamental sense, ideas make growth and development possible."
> *ibid, p. 68; italics added*.

[x] Obviously, the book must have an 'appendix' instructing the user on the necessity and mode of using the *axiom of choice*. Every indiscriminate reliance on *indexing over a continuum of agents, technologies* etc., is an implicit appeal to the axiom of choice, or one of its *noneffective* and *noconstructive* equivalents.

The final metaphoric invocation is to get hints on the way to encapsulate, formally, the role played by ideas, defined as *evolving bit-strings*, when 'used to produce human capital'. Here Romer relies on neurophysiological metaphors: ideas, literally, reconfigurate the architecture of the neural network representation of what Simon would term the Thinking (Wo)Man. 'Ideas.....represented as pure pieces of information, as bit strings' (p. 71) enhance the productivity of physical capital solely by a rearrangement of the possible permutations of the constituent elements that go into its manufacture: be it a process, such as sewing a shirt, or a piece of equipment, say a computer. Similarly, they enhance the value of human capital by reconfiguring the physical architecture underlying, say, thought processes:

> "Now consider human capital. In my brain there are different physical connections between my neurons. ....[T]he knowledge that reading a software manual [for a new computer and new word-processing software gives] rearranges connections in my brain and makes my human capital more valuable. .... The increased value is created by new ideas. Whether it takes the form of a hardware design, software code, or an instruction manual, an idea is used to mix or arrange roughly the same physical ingredients in ways that are more valuable. And in each case, these ideas can be represented as pure pieces of information, as bit strings".
> *ibid, p. 71*

Once again, therefore, ideas represented as bit strings encapsulating 'pure pieces of information'[xi] function as inputs into a physical architecture representing human capital and transform its 'wiring', so to speak, in such a way that 'it' is able to process them more effectively, in some sense. From standard results in automata and computability theory, it is well known that neural network architectures can be given recursion theoretic formalisms as automata of varying degrees of complexity. To be consistent with the standard postulates of rationality in economic theory it is, however, necessary to postulate an architecture that is formally equivalent to a Turing Machine. Such an architecture allows rational decision processes to exhibit a kind of formal untamability of ideas. Let me expand on the heuristics of this last comment a little more (to supplement the previous discursive comments).

The inadequacy of the traditional *book of blueprints* vision of feasible technologies becomes patently evident if any such interpretation is attempted for ideas held by rational economic agents interpreted as Turing Machines. Even if the neurons in a brain are finite, not even the proponents of *strong AI* would suggest that the world of ideas in any unit can formally be

[xi] The ideal way to proceed, at this point, would be to interpret and define information also recursion theoretically, for which there is a well developed tool-kit provided by ***algorithmic information theory***.

tamed or accessed - unless by magic or the kind of sleight of hand involved in invoking the *axiom of choice*. Somehow, somewhere, the open-endedness of ideas must assert itself in some kind of indeterminacy *in models of growth and development*. That is why the past can never hold all the secrets to the future. Trivial though this remark may sound, to formally encapsulate it in an interesting and operational way is not easy.

A recursion theoretic formalism

I can, now, put together a recursion theoretic formalism. Before doing this it is necessary to summarize Romer's production sub-model . Romer considers output, *Y*, to be an additive function of a standard production function and a term representing the production of ideas, one for each of, say, *n* sectors as follows:

$$Y = F(K, L, T) + \sum_{j=1}^{n} Gj(Kj, Lj, Hj; Aj) \qquad (1)$$

In addition to the conventional notation we have:
Hj: human capital used in sector (or activity) *j*;
Aj: 'idea' characterizing sector (or activity) *j*;

The search for new ideas is formalized as a general dynamical system as follows:

$$A(t+1) = S[H_A(t), (A,(t), A_2(t)....., A_N(t)), (H,(t), H_2(t),...., H_n(t))] \quad .... (2)$$

This has the following interpretation; the genesis of new ideas is a function of:

$H_A(t)$: human capital used exclusively in the search of the 'space' of ideas at time *t*;

$A_i(t) : i = 1,.... N(n \le N)$, the collection of ideas available in a specified economic region at time *t*;

The role of ideas in enhancing human capital and a learning-by-doing specification can together be captured in the following way to complete the output submodel:

$$H(t+1) = \Omega[H(t), A(t)] \quad .... (3)$$

Now, according to the intuitive definitions:

(I) $A_i(t), (\forall i = 1, \ldots N)$, are specified as bit strings;

(II) $H_j(t), (\forall j = 1, \ldots n)$, when considered as arguments of $G_j(\forall j = 1, \ldots, n)$, are neural networks; to be consistent with the rationality postulates of economic theory, these neural networks must be capable of *computation universality*;

Then, by the *Church-Turing Thesis*, we can represent each $H_j, j = 1,\ldots n$ and H_A as programs (or algorithms) computationally equivalent to the corresponding Turing Machine. Next, by stacking the bit strings $A_i, \forall i = 1,\ldots, N$, we can consider the prevailing collection of ides as a program (or algorithm)[xii]. This means the arguments of the function *S* in (2) are a collection of programs and, thus, search can be said to be conducted in the space of programs. At this point a direct genetic programming interpretation of the (computable) search function *S* makes the dynamical system (2) naturally evolutionary. However, the bitstring representing ideas can be retained as the data structures for the programs, partial recursive functions and Turing Machines in (1)-(3). Then, search will be conducted in the space of programs and data structures.

A similar interpretation for (3) is quite straightforward. However, (1) is an entirely different matter. Standard definitions define the arguments of *F* and K_j and L_j as arguments in G_j on the domain of *real numbers*. Given the algorithmic definitions of H_j and A_j, it is clear that G_j must be a partial recursive function for the whole system (1)-(3) to make analytic sense. This means one of two possible resolutions.

(a). Either, K_j and L_j $(\forall_j = 1,\ldots,n)$ must be defined as *computable reals*; hence, extended re-definitions of the domain of definition of H_j and A_j from the *computable numbers* to the *computable reals*;

(b). Or, K_j and L_j defined over the (countable set of) computable numbers;

Either way standard constrained optimization must be replaced by combinatorial optimization, on the one hand; and, on the other hand, one loses the applicability of separating hyperplane theorems[xiii] and, hence, welfare and efficiency properties of equilibria cannot, in general, be derived by algorithmic methods.

I shall conclude this section with two formal propositions.

[xii] Recall that '[A]n idea is the increment in information that comes from sorting ....' . In this connection see the illuminative discussion in Nelson and Winter (1982, ch. 4, §1) on 'Skills as Programs' and Bronowski's 'Silliman Lectures' (Bronowski, 1978, ch. 3).

[xiii] More generally, the Hahn-Banach theorem.

Proposition 1

Given the recursion theoretic interpretation of (2), there is no *effective procedure* (i.e., *algorithm*) to 'locate' an arbitrary Pareto improving configuration of ideas from the given configuration as an initial condition.

Proof

The proof of this proposition is based on a simple application of the Rice or Rice-Shapiro theorems in classical recursion theory. The dynamical system that is (2), given the recursion theoretic interpretation, can be represented by an appropriate *Universal Turing Machine* or, equivalently, by an appropriate *Universal Program*. The given initial condition for the dynamical system (2) corresponds to initial configurations for a Turing Machine computation and its program equivalent. These initial conditions and configurations correspond, economically, to the *status quo* set of ideas. But by Rice's theorem no nontrivial subset of programs can be effectively located by starting from any arbitrary configuration for a Turing Machine.

Remark

In other words there is no *a priori* local search *procedure* that can be used to discover a Pareto-improving set of ideas.

Proposition 2

Given an initial, empirically determined, configuration of ideas represented algorithmically in (2), there is no effective procedure to determine whether *S*, implemented as a Program - say as a *Genetic Program* - will halt (whether at a Pareto improved configuration or not).

Proof

The proof of this proposition is a trivial consequence of the *unsolvability of the halting problem for a Turing Machine*. The necessary contradiction is obtained by supposing that there is an effective procedure to determine whether any given empirically determined configuration of ideas, used as initial conditions to implement a program for a Turing Machine, will result in a well-defined set of output values.

Remark

In other words, this proposition suggests that it is impossible to find, by algorithmic means, definite answers to questions about the existence of feasible production processes to implement any given set of ideas.

These two propositions cast doubts on the blessings of the 'curse of dimensionality' to which Romer refers (cf. above and ibid, pp. 68-9). There are no effective procedures, discoverable *a priori* and systematically, to determine which 'small fraction of the large number of possible mixtures may be valuable'. This is why *economic development*, like the evolutionary paradigm itself, is so difficult to encapsulate in simplistic, formal, growth models - endogenous, or not.

§3. Towards an o Open-Ended Theory of Economic Development

> "[Tacit knowing is when] we can know more than we can tell. .....
> [S]uppose that tacit thought forms an indispensable part of all knowledge, then the ideal of eliminating all personal elements of knowledge would, in effect, aim at the destruction of all knowledge. The ideal of exact science would turn out to be fundamentally misleading and possibly a source of devastating fallacies. ....
> [A] mathematical theory can be constructed only by relying on prior tacit knowing and can function as a theory only within an act of tacit knowing, which consists in our attending from it to the previously established experience on which it bears. Thus the ideal of a comprehensive mathematical theory of experience which would eliminate all tacit knowing is proved to be self-contradictory and logically unsound."
> *Polanyi, 1966, pp. 4, 20-1; italics in original.*

The question, now, is how to embed Romer's enhanced combinatorial model within an *institutional* framework that is conducive to *development*. I suggest that Romer's *trained person* adds *The Tacit Dimension* (Polanyi, 1966, especially, ch.1, pp.1-27; but cf. also Polanyi, 1962, especially Part Two), among other things, to his enhanced growth model.

To that extent the model has to be formally open ended; i.e., with some indeterminacy. However, the indeterminacy is not arbitrary - there must be, proverbially, *some method in the madness*. The above two propositions are an attempt to encapsulate formal indeterminacy in a structured way. Some kind of formal border between what can be known, learned and 'told' - i.e., formally so described - and that which cannot be so described defines the dividing line between the neat and determined world of formal growth models and the messy and evolutionary development process. The skeletal recursion theoretic formalism and interpretation of Romer's *ideas* given above, and the ensuing two propositions, makes it possible to indicate the formal nature of this dividing line. In general, processes that are *recursively enumerable but not recursive* allow the kind of indeterminacy I am suggesting. The proofs of the above two propositions would locate the indeterminate range without actually determining them - to put it somewhat paradoxically.

It is interesting to note that Paul David (David, 1993), in a not-unrelated contribution, tackles the broader issue of the role, nature and scope of knowledge in technological change and, hence, in the growth processes of economies. He, too, proposes an interesting dichotomy for the definition of knowledge: *codified* and *tacit* (the latter along lines suggested by Michael Polanyi, op cit.[xiv]). These definitions are also based on an economic setting with an institutional structure that seems to have an algorithmic and evolutionary perspective. Paradoxically, and contrary to received wisdom, it is possible, I believe, to use the notion of *oracle* (or relative) computation[xv] to recursion theoretically formalize tacit knowledge. On the other hand, codified knowledge is straightforward algorithmic knowledge and almost identical, formally, to Romer's concept of idea.

Let me try to explain, in an elementary and heuristic way, the meaning of the above remarks. There is no better way to summarize Polanyi's pioneering attempts to delineate tacit from non-tacit knowledge than his concise but richly evocative statement that '*we can know more than we can tell*'. I have suggested that this statement encapsulates the role of Romer's 'trained person', to whom one must turn to implement production sequences that have somehow been transplanted from one institutional and historical setting to another. The essential point and role of Romer's 'trained person' and David's 'tacit knowledge' is that their expertise *cannot be formalised and transplanted*; but they are necessary for the operational part of production sequences to function 'efficiently'.

Assume, now, that the 'codified' part has been transplanted in the form of production processes, formalised, as suggested above, recursion theoretically. An operative, even as part of the formalised production process, may occasionally have to seek the 'trained person's' advice and help on effecting a particular decision at some point in the sequence. How can this role be 'formalised' in the recursion theoretic formalism I have employed above? I believe there is a simple answer although ***the simplicity belies its combinatorially complex content***. The simple answer is to embed the model in its standard recursion theoretic formalism within a framework capable of appealing to an 'oracle' for advice and help, as and when the need arises when nonrecursive problems are encountered.

[xiv] cf. also Nelson and Winter (1982, ch. 4, §2).

[xv] cf. Davis (1982, ch. 1) or Rogers (1967, ch. 9).

In other words, as ‘codified knowledge’ is implemented in the form of transplanted production processes formalised recursion theoretically, the relevant operative will seek the help of the ‘trained person’ whenever knowledge and skills that are ‘*known but cannot be told’* will be required. This category of knowledge can - and must - include patented knowledge as well. This is almost exactly analogous to a computation process which, from time to time, halts and requests additional, non-recursive information before it can proceed. Thus, the rational economic agent as a Turing Machine operating or implementing ‘codified knowledge’ of ideas formalised as ‘bit-strings’ will, on encountering the need for knowledge that could not or may not be so represented will appeal to the ‘oracle’ for help before proceeding with the computation, decision process and so on. The only non-formal requirement we will have to append here is that which is classically attributed to an *oracle.* Under this interpretation the standard model of oracle or relative computation is more than adequate for the purposes I have in mind.

There remains, finally, the thorny question of the role of public policy in fostering the search for ideas and the broadening of the knowledge-base of a country or region. The formalism based on Romer’s suggestions must be complemented by at least two kinds of case-historic examples: one, examples of the evolution of ideas in the basic sciences in their interaction with applied sciences.

Secondly, case-histories of the institutional underpinnings, and their evolution, for the development of science and technology in the economic growth process, underpinning economic development. I conjecture that a fertile area of study in this direction is an investigation of the institutional basis for the interaction between science and technology in Tokugawa and early-Meiji Japan - and the transition from one to the other. It is that transition regime, I conjecture, that is most relevant for the more dynamic developing economies, such as India, China and Brazil, for lessons on science and technology policy for development and growth.

Ideas for understanding transition regimes must, surely, be based on *non-algorithmic* processes. I suggest a way to make sense of this is to consider economic development as a combinatorial problem, to which a first step is suggested with the above formalism. The metaphor, going beyond Meccano, Chemistry and Chess Sets, is to use the ***Game of GO*** **–** in this difficult task.

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