On Innovation, Instability, and Growth

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I discuss a parsimonious stochastic framework of innovation and corporate dynamics, which encompasses the Gibrat's Law of Proportionate Effect and the growth process originally described by Herbert A. Simon as particular instances. The framework generates testable propositions along four different dimensions: (i) firm size distribution, (ii) the distribution of firm growth rates, and (iii-iv) the relationships between firm size and the mean and volatility of firm growth rates. The predictions of the stochastic benchmark are not falsified across different data sets. Both size and growth distributions are 'fat tailed', while innovation, competition, and instability at the level of products and firms map onto macroeconomic shocks at the aggregate level of Country GDP. First, the probability density function of growth rates appears to be invariant at different scales of observation and across periods, with its central body well approximated by a Laplace distribution and tails that contain many more events than predicted by a Gaussian probability density function and exhibit an inverse cubic power law. Second, since firm size distribution is fat-tailed, micro level shocks that affect large firms generate aggregate fluctuations, and GDP volatility remains substantial even for large diversified Countries. The proportional growth benchmark sketched in the paper suggests that the origins, magnitude, composition, and propagation of microeconomic shocks from business firms are key to understand the 'state of economy' and GDP growth and volatility.

1. Introduction

A recent strand of research has developed a new view on the origin of aggregate shocks within the economy, claiming that a significant fraction of aggregate fluctuations arise from idiosyncratic shocks to individual firms. "Many economic fluctuations are attributable to the incompressible grains of economic activity, the large firms. I call this view the 'granular' hypothesis. In the granular view, idiosyncratic shocks to large firms have the potential to generate nontrivial aggregate shocks that affect GDP, and via general equilibrium, all firms. The granular hypothesis offers a microfoundation for the aggregate shocks of real business cycle models (Kydland and Prescott (1982)). Hence, real business cycle shocks are not, at heart, mysterious "aggregate productivity shocks" or "a measure of our ignorance" (Abramovitz (1956)). Instead, they are well defined shocks to individual firms". (Gabaix, 2011, p. 735).

I refer to a stochastic benchmark developed jointly with H.E. Stanley, S. V. Buldyrev, and M. Riccaboni (see Fu et al. 2005; Growiec, Pammolli & Riccaboni, 2013; Riccaboni et. al., 2008, Buldyrev, Pammolli, Riccaboni & Stanley, 2013), which accounts for statistical regularities at different levels of aggregation within the economy. The benchmark represents business firm growth as the outcome of both the entry of new business opportunities through innovation and a continuous growth process of the size of products. This simple characterization is sufficient to demonstrate why observed idiosyncratic risk (volatility) at the firm level cannot be washed/diversified away and, on the contrary, it becomes relevant for macro economic outcomes and affects the amplitude of GDP fluctuations.

Since Robert Gibrat's Book *Les Inégalités Economiques*, published in Paris in 1931, the widespread presence of skew distributions in economic systems has been addressed, giving impulse to an influential strand of models (see Sutton, 1997, for a review). According to Gibrat's Law of Proportional Effect, skew distributions of a given variable x can be accounted for by postulating that some underlying function of x is normally distributed. In particular, Gibrat postulated that the logarithm of x developed accordingly, so that the expected value of the increment to a firm's size in a given period is proportional to the current size of the firm. In other words, the Gibrat's hypothesis assumes that the simplest continuous Markov chain, namely an unrestricted random walk, describes the dynamic behavior of the logarithm of the size and of the growth rate of firms. From the Central Limit Theorem, the probability distribution of firm size and firm growth rate are lognormal, but it's not possible to characterize an equilibrium probability distribution, since both the mean and the variance increase linearly over time. Such predictions are at odds with empirical regularities, and have been relaxed over time. In particular, Herbert A. Simon developed a discrete time stochastic model with field effects, which accounts for the existence of competitive interactions among firms, modeled through entry and exit of business opportunities (Ijiri, Simon, 1977). In Simon's framework, the Markov chain describing the dynamic behavior of firm size is a Polya urn, while entry and exit sustain the existence and stability of the equilibrium probability distribution, which is a Pareto.

Over time, Gibrat's Law has been combined with a set of ancillary assumptions (see Sutton, 1997; Luttmer, 2010) in the context of models in which the stochastic element has been often introduced into standard maximizing models.

However, known regularities refer to properties of stationary distributions and, individually, have little or no power to discriminate across broad classes of stochastic processes that might have generated them (see Brock, 1999). For example, any weak dependent process, under regularity conditions, satisfies the Central Limit Theorem and depicts a Gaussian stationary distribution. Such information is not sufficient to identify a specific generative process, but rather a class of processes with a given set of properties.

It is a claim of this paper that, in order to condition upon otherwise unconditional distributions, predictions of firm growth models must be tested simultaneously against multiple predictions and stylized facts.

Even though a set of empirical regularities has been repeatedly observed, a comprehensive framework on the size and growth of business firms has been only recently outlined (see Fu et. al, 2005; Growiec, Pammolli & Riccaboni, 2013).

First, it is well known that the size distribution of business firms is skewed. While the exact shape of the size distribution is still debated, the Pareto and lognormal distributions are typically retained as useful benchmarks (Stanley et al. 1996, Axtell 2001, Cabral & Mata 2003, Luttmer 2007, Growiec, Pammolli & Riccaboni, 2008 and 2013).

Second, the growth rate distribution is not Gaussian but 'tent-shaped' in the vicinity of the mean growth rate, and the generative mechanism behind belongs to the Exponential Power distributions family, which includes the Laplace and the Gaussian distributions as particular cases (Stanley et al. 1996, Pammolli et al. 2007). By looking at the entire distribution, it is possible to document how rare events of extremely large positive and negative growth shocks, induce power law tails in the firm growth rate distribution.

Third, it is true that the variance of growth rates is systematically higher for smaller firms (Hymer & Pashigian 1962, Mansfield 1962, Evans 1987). However, the variance of growth rates of both companies and countries decays with size as a power-law, with an exponent of about 1/5, indicating that volatility of large diversified companies does not decay according to $\sqrt{N}$ (Stanley et al. 1996; Bottazzi et al., 2001; Sutton 2002; Riccaboni et al. 2008; Gabaix 2011).

Fourth, smaller firms have a lower probability of survival, but those that survive tend to grow faster on average than larger firms. Among larger firms, mean growth rates are unrelated to past growth or to firm size (Mansfield, 1962).

The paper is organized as follows: In section 2 the stochastic benchmark is briefly outlined. Section 3 illustrates the predictions the benchmark on the shape of size and growth distributions, as well as on the relationship between size and growth volatility. Moreover, results of empirical investigations are discussed. Section 4 concludes.

2. A Stochastic Benchmark for Business Firm Growth

Several models of proportional growth have been developed introduced in economics to account for the size dynamics of business firms (for reviews see Sutton, 1997; Coad, 2009).

A general and parsimonious representation of processes of firm growth built upon two key sets of assumptions (see Figure 1), which recognize a complementarity between two different mechanisms underlying corporate growth:

A. The number of units (business opportunities) in a class (firm) grows in proportion to the existing number of units (Herbert Simon Preferential Attachment mechanism)
B. The size of each unit (business opportunity) grows in proportion to its size, independently of other units (Gibrat's Law of Proportional Effect)

The above assumptions can be stated as follows:

1. Each firm α consists of $K_\alpha(t)$ units. At time $t=0$ there are $N(0)$ firms of unitary size, so that the total number of units in the initial period is given by $n(0)=N(0)$. At each moment in time, there is a constant arrival rate μ of new units, and a constant destruction rate λ. The net arrival rate of new units $\psi \equiv \mu - \lambda$ is assumed to be positive. The number of units at time t is thus $n(t) = n(o) + \psi t$. Without loss of generality, φ is normalized to unity.
2. With birth probability $b \in [0,1]$, a new unit is assigned to a new firm. With probability 1-b, it is assigned to an existing firm. The probability that an incumbent firm α will capture a new unit at time t is proportional to the number of active units it has: $P_\alpha = \frac{(1-b)K_\alpha(t)}{n(t)}$

The two fundamental assumptions of the framework can be interpreted as follows: Larger firms can afford to invest more in R&D. Hence, large firms tend to capture more new business opportunities resulting in a larger flow of new products (see also Klette & Kortum 2004). Assuming proportional growth in the number of units per firm corresponds to postulate constant returns to scale in R&D. A positive rate of entry (b > 0) implies that a fraction of the R&D output is not generated by the incumbent firms. When an innovator not affiliated with a large company is successful, she starts up a new firm, which initially consists of a single unit selling this freshly innovated product. Later, it may as well grow and sell more products. Firm exit is not modeled here, but parameter b captures the net entry rate –entry minus exit –, which, in a growing economy, ought to be positive in the long run.

The second set of assumptions in the model is defined as follows:

3. At time t, each firm α has $K_\alpha(t)$ units of size $\xi_i(t)$, i=1,2,…,$K_\alpha(t)$ where $K_\alpha(t)$ and $\xi_i(t) > 0$ are independent random variables.
4. At time $t+1$, the size of each unit is decreased or increased by a random factor $\eta_i(t) > 0$ so that

$$\xi_i(t+1) = \xi_i(t) + \eta_i(t,) \quad (5)$$

   Where $\eta_i(t,)$, the growth factor of unit i, is a random variable that independent of all other η_i's and ξ_i's. It is assumed that $\eta_i(t,)$ is taken from a distribution $P_\eta \eta_i$ with finite mean $E\big(ln\, \eta_i(t)\big) \equiv m_n$ and finite variance $Var(ln\, \eta_i(t)) \equiv E(ln\, \eta_i(t))^2 - {m_n}^2 = V_\eta$. The size of the class is defined to be $S_i(t) = \sum_{j=1}^{k_i(t)} \xi_i(t)$.
5. The size of every new unit arriving at time t is drawn at random from the distribution of unit sizes. Its expected size is denoted as $\bar{\xi}_i(t)$. The growth rate of each class is then defined to be:

$$g_\alpha \equiv \left[\frac{S_\alpha(t+1)}{S_\alpha(t)}\right] \equiv log \sum_{i=1}^{K_\alpha} \xi_i(t+1) / \sum_{i=1}^{K_\alpha} \xi_i(y)$$

A few qualifications are needed. First, since unit sizes fluctuate independently of each other, any unit occupies a well-defined market niche. Second, by assuming multiplicative fluctuations, we assume that demand shifts affect all units proportionately, and that the variance of growth rates is independent from size. Third, by assuming that units cannot move between firms, the model implies the existence of sunk costs and human/organization capital necessary for production (Luttmer 2010), whose transfer between firms is too costly to occur. Finally, the framework postulates that increases in the size of existing units are independent from the arrival of new units. In other words, the average size and number of units within a firm are assumed to be independent.

The economic rationale behind the above set of assumptions is straightforward. First, in a growing economy, one should expect the average net growth rate of unit sales to be positive, in line with the macroeconomic 'stylized facts'. Coherently, we assume that $m_\eta \geq 0$. Nonetheless, that condition does not preclude the Schumpeterian process of creative destruction, an obsolescence effect, or the existence of product life cycles.

Second, the assumption that new units are, on average, proportional in size to the already existing units, is meant to capture the disembodied component of technical change. If the overall rate of technical progress is positive ($m_\eta \geq 0$), then it is natural to expect that not only existing units, but also new opportunities will benefit from it. Otherwise, new units would become increasingly smaller in proportion to the established ones, and the average age of units would become the crucial factor behind firm size – an assumption, which is at odds with evidence.

The framework generates testable predictions along multiple relevant dimensions, i.e. the distribution of growth rates, the size distribution, and the relationship between size and volatility of growth rates. This is an important feature, in order to test the plausibility of the dynamics of the generative stochastic process, which is hypothesized.

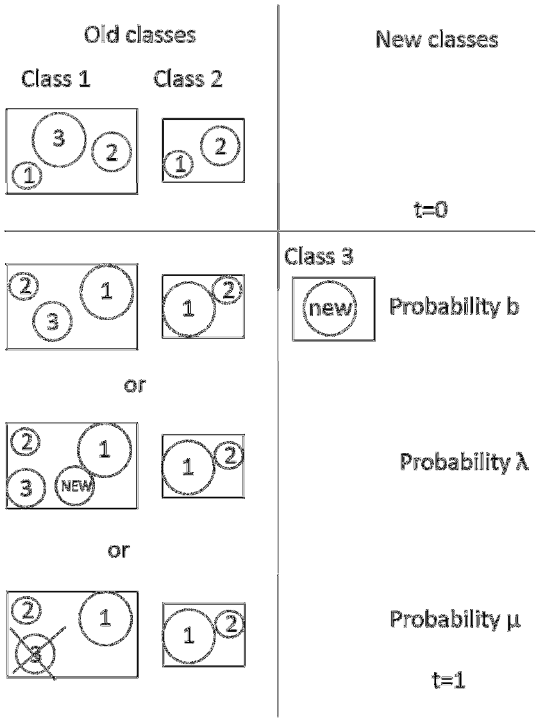

Fig 1. A stochastic benchmark for business firm growth. At time t=, there are N(0)=2 classes (□) and n(0)=5 units (○) Assumption (1). The area of each circle is proportional to the size ξ of the unit, and the size of each class is the sum of the areas of its constituent units (assumption 4) At time t=1 a new unit is created or deleted. With probability b the new unit is assigned to a new class (Class 3 in the figure) (Assumption 2). The size of the unit is taken from the distribution of the existing units (assumption 5). With probability λ the new unit is assigned to an existing class with probability proportional to the number of units in the class (assumption 2). In this example a new unit is assigned to class 1 with probability 3/5 or to class 2 with probability 2/5. With probability μ a randomly selected unit is deleted.

The size distribution

As mentioned above, according to the Gibrat's law of proportional growth, firm growth rates are realizations independent of size, and the size distribution is a lognormal, while according to Herbert A. Simon and coauthors, it is well approximated by a Pareto distribution, at least in the upper tail (Simon, 1958: Ijiry and Simon, 1977).

Our benchmark accounts for the shape of size distributions for products and firms. In particular, Growiec, Pammolli & Riccaboni (2008) have shown that the distribution at the micro level of products is lognormal, while aggregation at the firm level leads to the emergence of a Pareto tail. In a nutshell, coherently with the predictions of the benchmark, the firm size probability density function is given by a lognormal distribution multiplied by a stretching factor, which increases with S. For very small values of S, the stretching factor becomes negligible and the distribution is close to a lognormal. The larger S, the higher is the influence of the stretching factor. The complementary cumulative density function P(S) for the size is given, for $t \to \infty$, by:

$$P(S) = -P'(S) = \underbrace{h'(S)}_{\text{Lognormal}} \times \underbrace{\sum_{k=1}^{\infty} \frac{h(s)^{k-1}}{k} (1-(k+1)e^{-k})}_{\text{Stretching factor}}, \tag{4}$$

Where $h(s) \equiv \phi\left(\frac{\ln\left(\frac{S}{K^{\gamma}}\right) - m_s}{\sqrt{V_s}}\right)$ and $\gamma \in [0,1]$.

The growth distribution

According to the model outlined above, it has been shown (Fu et al., 2005) that the growth distribution is given by:

$$P_g(g) \equiv \sum_{k=1}^{\infty} P(K) P_g(g|K),$$

Where P(K) is the probability density function (PDF) of the number of units in the classes. The conditional distribution of the logarithmic growth rates $P_g(g|K)$ for firms consisting of a fixed number of units converges to a Gaussian distribution for $K \to \infty$. In particular:

$$P_g(g|K) \approx \frac{\sqrt{K}}{\sqrt{2\pi K}} \exp[(g-\bar{g})^2 K/2V],$$

Where V is a function of parameters of the distribution $P_\xi(\xi)$ and $P_\eta(\eta)$ and $\bar{g}$ is the logarithm of the mean growth rate of a unit.

When entry of new units and new classes is considered, the probability density function (PDF) of the number of units per class (P(K)) and the PDF of the growth rates can be derived (see Fu et al. 2005). The expression for P(g) can be computed analytically only in some limiting cases.

As time t goes by, the growth rate distribution gradually comes to exhibit power-law wings. Finally, for $t \to \infty$, it is approximated by:

$$P(g) = \frac{1}{1-b}\frac{1}{\sqrt{2\pi V}}\int_0^{+\infty} e^{-y} y^{\frac{1}{1-b}} \left(\int_0^{+\infty} e^{-\frac{(g-\bar{g})^2 K}{2V}} K^{-\frac{1}{2}-\frac{1}{1-b}} dkK\right) dy$$

Which, when $b \to 0_+$, can be written as:

$$P(g) \approx \frac{2V_g}{\sqrt{g^2 + 2V_g}\,(|g| + \sqrt{g^2 + 2V_g})^2} \quad (1)$$

The distribution in Equation 1 combines a double exponential body for small values of g and power law decay for large values of g.

The distribution P(g) could be obtained also as a scale mixture of Gaussians (see West, 1987 and Andrews & Mallows, 1974 where the mixing distribution (P(K)) is exponential. The pdf of growth rates is given by:

$$p(g) = \int_0^{\infty} \lambda e^{-\lambda K} \frac{1}{\sqrt{2\pi V K^{\psi}}} exp\left\{-\frac{g^2}{2VK^{\psi}}\right\} dK \quad (3)$$

Where ψ is the scaling parameter. In a nutshell, we have a generalization of a Gibrat's growth process amplified by an event that may occur with probability K and may change the probabilistic structure of the stochastic process (see Kotz et al., 2001; Pammolli et al. 2007; Growiec, Pammolli & Riccaboni, 2013).

Parameter ψ captures the scaling of the variance and reflects the fact that for different time frames the strength of dependence of firm size distribution on the number of its products varies. Indeed, for short time frames (few products), one should expect a strong dependence and thus a high ψ (due to a small number of idiosyncratic product specific shocks), while ψ falls with the length of the time frame.

The predictions of the model have been tested analyzing a variety of data sets, at different levels of aggregation of economic systems, from micro level of products and firms, to macro level of industrial sectors and national economies. Evidences in Figures below, as well as accurate econometric testing in Pammolli et al., 2007, show that the predictions of the stochastic framework are not falsified by empirical evidence.

Surprisingly (but coherently with the predictions of the outlined benchmark), fluctuations exhibit invariant statistical properties at different levels of aggregation (Figures 2 and 3), since a broad central region of the distribution of annual growth rates is well approximated by a double exponential distribution (Laplace distribution, see Kotz et al., 2001), while the tails of the probability density function of growth rates exhibit, at any level from products to GDP, an inverse cubic power law. In other words, the pdf of growth rates contains many more events in the tail than predicted by the Gaussian pdf, which would classify them as outliers.

This scale invariant property is important, since it indicates that a scale free function seems to characterize the underlying dynamics.

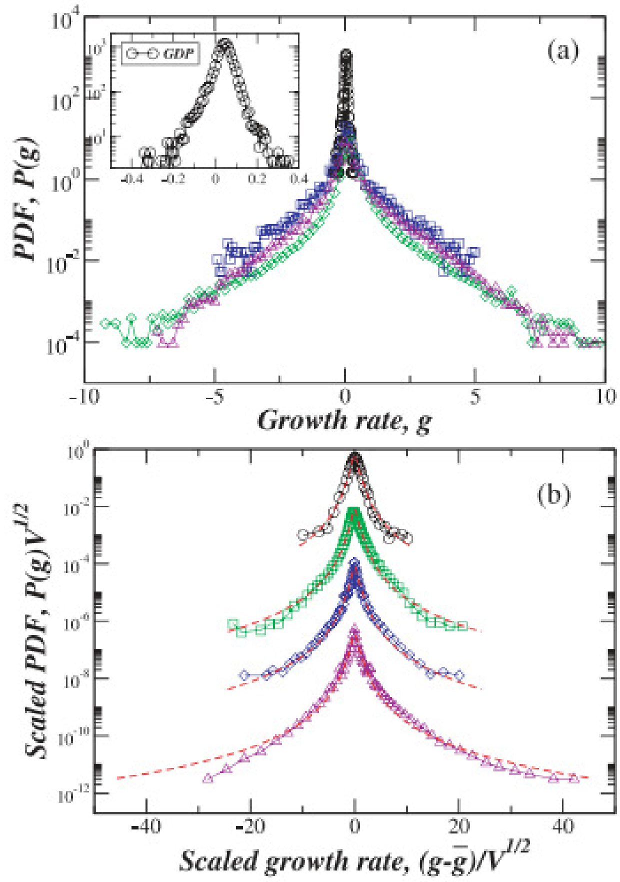

Fig. 2 (a) Empirical results of the probability density function (PDF) p(g) of growth rates. Shown are country GDP (○), pharmaceutical firms (□) manufacturing firms (◊) and pharmaceutical products (Δ). (b) Empirical test of equation (1) for the probability density function P(g) of the growth rates rescaled by $\sqrt{V}$. Dashed lines are obtained based on equation (1) with V≈4x10^{-4} for GDP, V≈0.014 for pharmaceutical firms, V≈0.019 for manufacturing firms, and V≈0.01 for products. After rescaling, the four PDFs can be fit by the same function. For clarity, the pharmaceutical firms are offset by a factor of 10^2, manufacturing firms by a factor of 10^4 and the pharmaceutical products by a factor of 10^6 .

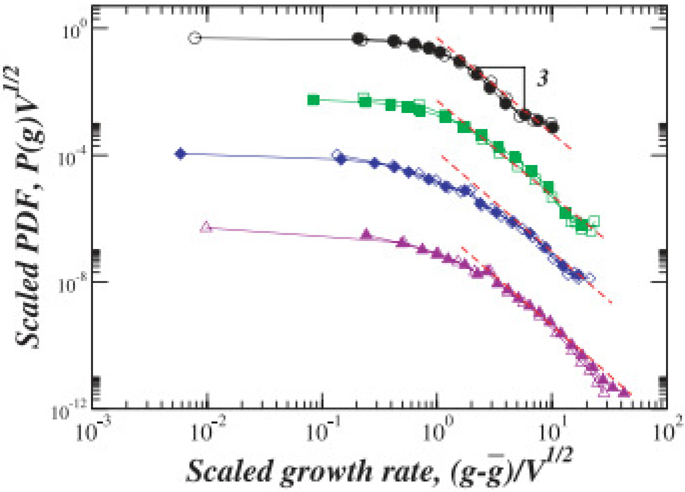

Fig. 3 Test of equation (1) for the tail parts of the PDF of growth rates rescaled by $\sqrt{V}$. The asymptotic behavior of g at any level of aggregation can be well approximated by power laws with exponent ζ≈3 (dashed lines). The symbols are the follows: country GDP (left tail:○) pharmaceutical firms (□), manufacturing firms (◊), pharmaceutical products (Δ)

The size-variance relationship

The relationship between the size and the variance of growth rates was originally investigated by H.E. Stanley and coauthors, who found that the size-variance relation follows an approximate power-law behavior, for both firms and countries, with a "power law coefficient fluctuating between -0.15 and -0.21 (Stanley et al., 1996).

The size-variance relationship crucially depends on the partition of firm (country) size across constituent components. If firms have P(K) units and V_η=0, for the Law of Large Numbers, σ(K)≈ K^β, where β =1/2. On the contrary, if each firms consists of a single unit only and V_η=>0, =0. When both mechanisms are at work, the speed of the crossover depends on the skewness of P(K). At one extreme, if all entities have the same number of units, β=0 and there is no crossover. On the contrary, if P(K) is power-law distributed, for a wide range of empirically plausible V_η, β is far from 1/2 and statistically different from zero. The size-variance relationship is not a true power law with a single well-defined exponent β, but undergoes a slow cross over from β=0 for S→0, to β=1/2 for S→ ∞.

The stochastic benchmark sketched above treats firms as classes composed of various number of units of variable size, can explain this size-variance dependence. In general it predicts that the size-variance relationship is not a true power law with a single well-defined exponent β, but undergoes a slow cross over from β=0 for $S \to 0$ to β=1/2 for $S \to \infty$ (see Riccaboni et al., 2008).

In a nutshell, we find that in the case of skewed size distributions, the size-variance relationship scales with the share of the largest constituent unit.

The predictions of the model have been tested in real world and simulation settings. The results of the tests are shown in fig. 4 and 5. The empirical conditional distribution $P_g(g|K)$ and the dependence of its variance σ^2 on K (the number of products) have been investigated. The coefficient for the power law exponent (-0.28) is significantly smaller than -0.5.

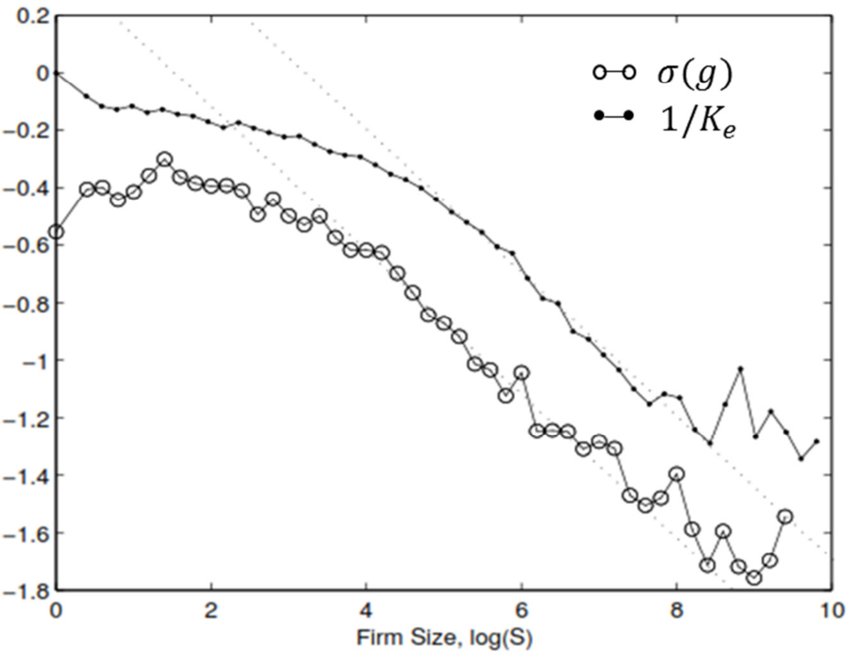

Fig. 4 Size-Variance relationship. The standard deviation of firm growth rates (σ) (circles), and the share of the largest products ($1/K_e$) (squares) versus the size of the firms (*S*). For $S < S1 = \mu_\xi \approx 3.44$, $\beta \approx 0$. For $S > S1$, β increases but never reaches 1/2 because of the slow growth of the effective number of products (K_e). The flattening of the upper tail is due to some large companies with unusually large products. A reference line whit slope 1/5 is also reported.

As shown in Figure 4 and Figure 5, the variance does not scale in agreement with the predictions of the Central Limit Theorem and, on the contrary, it scales with the share of the firm's largest unit. In our empirical investigations at the level of industries, the size-variance scaling coefficient β is found to be ≈ 1/5. In particular, we can observe that the coefficient for the power law exponent (-0.28) is significantly smaller than -0.5. Figure 5 shows the simulations results in the case of skew distributions of products size $P_\xi(\xi)$ (lognormal) characterized by la large variance. Under that assumption, the convergence of $P_g(g|K)$ to its Gaussian limit is extremely slow. As a consequence, the growth rates of the firms are determined by the growth rate of a few large products. Using the empirical values ($\mu_\xi = 3.44, V_\xi = 5.13, \mu_\eta = 0.016, V_\eta = 5.13$) and assuming the log normality of the distributions $P_\xi(\xi)$ and $P_\eta(\eta)$, it can be proved (see Riccaboni et al., 2008) that the behavior of σ is well approximated by a power law $\sigma \sim K^{-0.20}$ for $k>10^3$. For this set of parameters, the convergence of the conditional distribution $P_g(g|K)$ to a Gaussian distribution is obtained only for $k>10^5$.

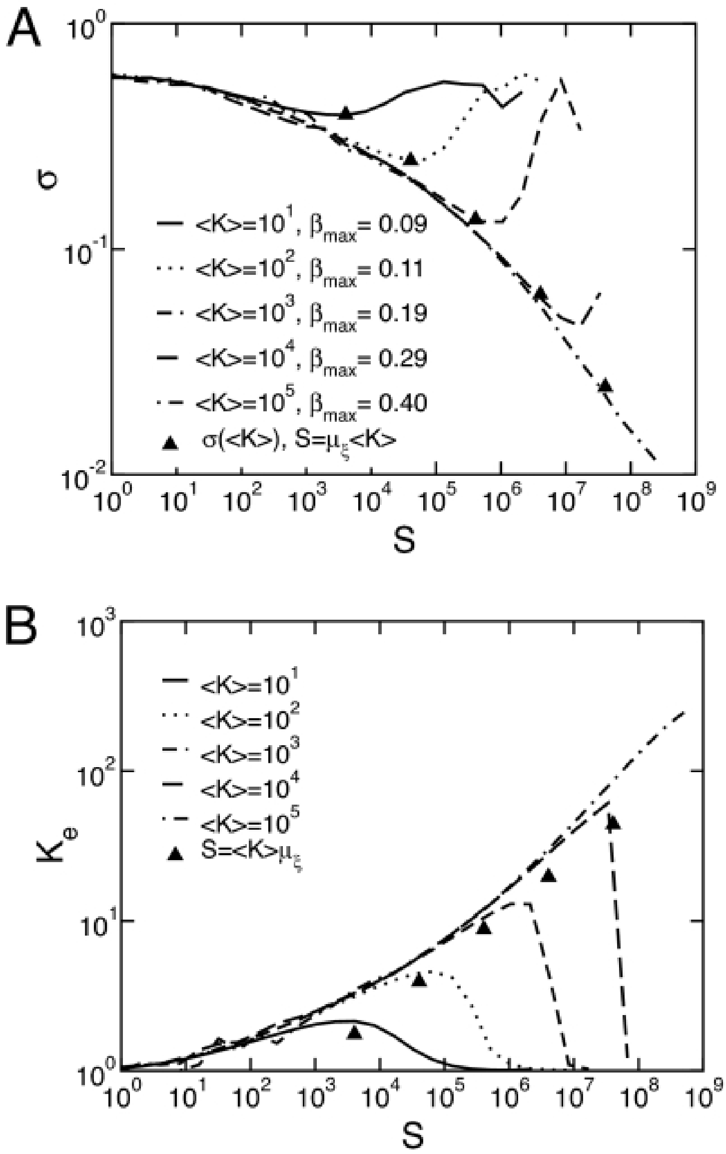

Fig. 5. Simulation results for the size variance relationship and the number of units. (*A*) Simulation results of the stochastic benchmark for σ(*S*) for *P*(*K*) = exp(−*K*/*K*0)/*K*0 with *K*0 = 10,102,103,104,105 and P_ξ and P_η lognormal with V_ξ = 5.13, m_ξ = 3.44, V_η = 0.36, μ_η = 0.016 computed for a database covering all the products commercialized by all the firms active within a specific industrial sector, worldwide. For small enough *S* and for different K_0, σ(*S*) follows a universal curve that can be well approximated with σ(*KS*), with *KS* = S/μ_ξ ≈ S/405. For $K_S > K_0$, σ(*S*) departs from the universal behavior and starts to increase. This increase can be explained by the decrease of the effective

number of units $K_e(S)$ for the extremely large firms. The maximal negative slope β_{max} increases as K_0 increases in agreement with the predictions of the Central Limit Theorem. (*B*). $K_e(S)$ reaches its maximum at approximately $S \approx K\mu_\xi$. The positions of maxima in $K_e(S)$ coincide with the positions of minima in $\sigma(S)$.

4. Discussion: Innovation, Instability, and Growth in Economic Systems

A stochastic benchmark of proportional growth in number and size of elementary units sheds light on the relationship among key interdependent features of business firms growth.

The benchmark concurs to explain why innovation and competition at the product and firm levels are key drivers of macroeconomic shocks and aggregate fluctuations.

Other authors (Carvalho, 2010; Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi 2012) adopt a network perspective and analyze sectoral interdependencies to show that "the presence of hub-like general-purpose inputs can undo the law-of-large-numbers argument and, thus, enables microeconomic shocks to affect aggregate volatility" (Carvalho and Gabaix, 2010).

The sketched stochastic benchmark provides a stylized representation of how capabilities and technological progress induce growth through an expansion of output varieties.

The number, relative size, and fluctuations of elementary constituent components that are loosely interdependent through entry and exit dynamics, create a fat-tailed distribution for firm growth rates, with a relatively high frequency of sudden, large changes in growth rates of individual companies (see also Simon 1955, Gabaix 1999, Luttmer 2007). Then, in turn, large firms or sectors out of that fat-tailed distributions shape GDP fluctuations[1].

The evidences presented in this paper support the view that microeconomic shocks induced by innovation and competition, together with the microeconomic composition of the economy, in terms of capabilities, diversification opportunities, and relative size of constituent components (sectors, firms, products) are are not diluted by aggregation and, on the contrary, are key to understand GDP volatility.

Additional information can be conveyd by an extension of the benchmark, in which product, firm, and sector life cycles, both within and across countries, are explicitly considered to understand aggregate volatility in its time varying dimension.

[1] Riccaboni and Schiavo (2010) have shown that the benchmark accounts also for the volatility observed in trade fluctuations across countries (see also Di Giovanni and Levchenko, 2012;).

This proposition deserves to be further investigated over time and across Countries, to study the interplay between microeconomic shocks, firm level capabilities, and other relevant factors, i.e., policies and institutions, taxation, etc. Moreover, the simple stochastic framework that we have outlined can be used to simulate the impact of propagation and amplification mechanisms, for example in relation to the relative size of the financial sector, or to quantify turnover and volatility specific to different sectors. Simulative exercises aimed at measuring the impact of external and internal amplifying mechanisms, can help to unravel the fragility and instability of different economic systems. Specifically, it is true that nonconvexities in the innovation process sustain technological diversification of richer countries, reducing their aggregate and idiosyncratic volality (Lucas, 1988; Koren & Tenreyro, 2013). However our results on the size variance relationship at different levels of aggregation show that, when growth and size distributions are skewed, the impact of diversication is significantly lower than predicted by the Central Limit Theorem in the case of an economy made by a large number of independent units of equal size (Hymer & Pashigian, 1962). While it is true that poorer and less diversified countries are more volatile in their growth performances, as the standard deviation of growth rates declines with the number of sectors/markets in which countries are active, volatility of large diversified countries does not scale as the square root of the number of sectors, since GDP growth is affected by the idiosyncratic shocks that affect a limited number of large companies and/or sectors.

The evidence and the stochastic framework outlined show that level capabilities, and industry structure matter for the 'state of the economy'. GDP growth and volatility regimes can be identified assessing the dependence of an economy upon a limited number of sectors or companies, as well as its susceptibility to idiosyncratic technological and demand shocks or, alternatively, the consequence of risk aversion and lack of investment in capabilities accumulation and escalation by undiversified entrepreneurs and companies (Hall & Woodward, 2010; Sutton, 2012).

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